Quaternion Gravity Sketch

Results from 1905

1. Special Relativity - Invariants of Nature

$dt^2 - dR_i^2/c^2 = d\tau^2$ - the interval $\qquad i = 1, 2, 3$

$E^2 - P_i^2 c^2 = m^2 c^4$ - mass

2. The Photon hypothesis

$E = h\nu \qquad P_i = \frac{h}{\lambda_i}$

Alternative year 1906: Instead of Riemann Geometry, use quaternions

1. Beat de Broglie

$(E, P_i c)^2 = (h\nu, \frac{hc}{\lambda_i})^2 = (m^2 c^4, E P_i c) \qquad m \neq 0$

2. Propose a new invariant of Nature

$(dt, dR_i/c)^2 = (dt^2 - dR_i^2/c^2, \underbrace{\boxed{2\, dt\, dR_i/c}}_{\text{Space-times-time}})$ ← what if this is invariant? Explore...

General case

$(u\,dt, V_i\,dR_i/c)^2 = (u^2 dt^2 - V_i \frac{dR_i^2}{c^2}, 2\,dt\,dR_i)$ if $u = \frac{1}{V_i}$

Could the invariance of space-times-time be about gravity?

Investigate: $u = f\left(\frac{M}{R}\right)$ - for simple spherically symmetric, non-rotating source mass

$u \to 1$ as $M \to 0, R \to \infty$.

To be consistent with weak gravity field tests: $u = e^{-\frac{GM}{c^2 R}}$ $V_i = e^{+\frac{GM}{c^2 R}}$

$(\underbrace{e^{-2\frac{GM}{c^2R}} dt^2 - e^{+2\frac{GM}{c^2R}} dR_i^2/c^2}_{\text{Rosen metric terms}}, \underbrace{2\, dt\, dR_i/c}_{\text{invariant space-times-time}})$

Quaternion gravity space-times-time could bend light the observed amount.

Quaternion Gravity Equations of Motion

The energy-momentum in spherical coordinates in the equitorial plane.

$$\left(\frac{E}{c}, P_R, P_\theta, P_\phi\right) = m\left(c e^{-\frac{GM}{c^2R}} \frac{dt}{d\tau}, e^{+\frac{GM}{c^2R}} \frac{dR}{d\tau}, 0, R\frac{d\phi}{d\tau}\right) \quad \text{eq. 1}$$

Square eq. 1:

$$\left(m^2c^4, 2EP_R/c, 0, 2EP_\phi/c\right) =$$

$$m^2\left(c^2 e^{-2\frac{GM}{c^2R}}\left(\frac{dt}{d\tau}\right)^2 - e^{+2\frac{GM}{c^2R}}\left(\frac{dR}{d\tau}\right)^2 - R^2\left(\frac{d\phi}{d\tau}\right)^2, c\frac{dt}{d\tau}\frac{dR}{d\tau}, 0, cRe^{-\frac{GM}{c^2R}}\frac{dt}{d\tau}\frac{d\phi}{d\tau}\right) \quad \text{eq. 2}$$

Focus on the first term, isolate $\frac{dR}{d\tau}$

$$\left(\frac{dR}{d\tau}\right)^2 + R^2 e^{-2\frac{GM}{c^2R}}\left(\frac{d\phi}{d\tau}\right)^2 = \left(c e^{-\frac{GM}{c^2R}}\frac{dt}{d\tau}\right)^2 - c^2 e^{-2\frac{GM}{c^2R}} \quad \text{eq. 3}$$

Eq. 1 has no dependence on time t or angle ϕ. Thus energy E and angular momentum L are conserved. Substitute from eq. 1 to eq. 3:

$$\left(\frac{dR}{d\tau}\right)^2 + e^{-2\frac{GM}{c^2R}}\frac{L^2}{R^2} = c^2\left(\frac{E^2}{mc^2} - e^{-2\frac{GM}{c^2R}}\right) \quad \text{eq. 4}$$

Keep the first order terms of the exponential $\left(\frac{GM}{c^2R}\right)$

$$\boxed{\left(\frac{dR}{d\tau}\right)^2 + \frac{L^2}{R^2} - 2\frac{GML^2}{c^2R^3} - 2\frac{GM}{R} = c^2\left(\frac{E^2}{mc^2} - 1\right)} \quad \text{eq. 5}$$

This is the same equations of motion as the Schwarzschild solution of general relativity. The quaternion gravity proposal will therefore pass test such as the precession of the perihelion of Mercury. At higher order in $\frac{GM}{c^2R}$, the series expansion are not identical. For light bending around a source, quaternion gravity predicts ~6% more bending (2nd order PPN accuracy).